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The Lee-Weinberg Bound Revisited

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Abstract

We report on a detailed numerical calculation of the minimum mass of stable, massive neutrinos in order that the contribution of the primordial neutrinos not exceed allowable limits on the present mass density of the Universe. We find limits $m_\nu \gtrsim 1.3 - 4.2$ GeV for Dirac neutrinos, and $m_\nu \gtrsim 4.9 - 13$ GeV for Majorana neutrinos, in qualitative agreement with previous estimates.

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One of the most important results in particle cosmology is the limit of the mass of stable neutrinos.[1] If a neutrino is more massive than a few hundred eV, it must be more massive than a "few" GeV. This latter limit is usually quoted as 2 GeV, and referred to as the "Lee-Weinberg" bound. The "Lee-Weinberg" bound was discovered independently by many people, with slightly different limits: Lee and Weinberg[2] ($m > 2$ GeV); Hut[3] ($m > 3$ GeV); Sato and Kobayashi[4] ($m > 1$ GeV); Vysotskii, Dolgov and Zel'dovich[5] ($m > 2.5$ GeV); Dicus, Kolb and Teplitz[6] ($m > 10$ GeV). Because of the importance of this limit, we present a detailed, numerical calculation of the contribution of massive neutrinos to the present mass density of the Universe, and reexamine the concommitant bounds on the neutrino mass.

The evolution of the neutrino number density is given by[7]

$$\dot{n}_\nu = \sigma_A |v| \left(n_\nu^2 - n_{eq}^2\right) - 3Hn_\nu , \tag{1}$$

where σ_A is the annihilation cross section, $|v|$ is the relative velocity, n_{eq} is the equilibrium number density,[8] and H is the expansion rate of the Universe, given in terms of the energy density ρ by (for the times of interest the curvature term was unimportant, and the Universe was radiation dominated)

$$H^2 \simeq \frac{8\pi G\rho}{3} \tag{2a}$$

$$\rho = \frac{\pi^2}{30} g_* T^4 \tag{2b}$$

where g_* counts the effective contribution of all the particles present in thermal equilibrium, $g_* = \Sigma\, g_{Bosons} + (7/8)\ \Sigma\, g_{Fermions}$. We define the dimensionless ratios $x = m_\nu/T$ and $Y = n_\nu/s$, where s is the entropy density given by

$$s = \frac{2\pi^2}{45}\, g_* T^3 \ . \tag{3}$$

In terms of x and Y, Eq. (1) is

$$\frac{dY}{dx} = \frac{-M\ M_{pl}}{x^2}\left[\frac{g_*\pi}{45}\right]^{1/2} \sigma_A |v|\ (Y^2 - Y_{eq}^2), \tag{4}$$

where $G^{-1/2} = M_{pl} = 1.2\times10^{19}$ GeV and $Y_{eq} = n_{eq}/s$.

We will consider two cases for $\sigma_A|v|$. The first case is for the annihilation of Dirac neutrinos. With the interaction term of neutrinos with fermions f_i of the form

$$\frac{G}{\sqrt{2}}\ \bar{\nu}\ \gamma_\mu\ (1-\gamma_5)\nu\ \bar{f}_i\ \gamma_\mu\ (C_{V_i} - C_{A_i}\gamma_5) f_i \ , \tag{5}$$

the annihilation cross is

$$(\sigma_A|v|)_D = \frac{G^2M^2}{2\pi} \sum_i\ (1-z_i^2)^{1/2}\ [(C_{V_i}^2 + C_{A_i}^2) + \frac{1}{2} z_i^2\ (C_{V_i}^2 - C_{A_i}^2)]\ , \tag{6}$$

where the sum is over all fermions with mass m_i less than M, $z_i = m_i/m$, and C_V and C_A are defined in terms of the electric charge and Z-component of weak isospin: $C_A = j_3$; $C_V = j_3 - 2q \sin^2\theta_w$ (e.g., for up quarks $C_A = 1/2$, $C_V = 1/2 - (4/3) \sin^2 \theta_w$). The relevant fermions included in the sum are leptons - N_ν light neutrinos, e, μ, τ; and quarks - u, d, s, t, b. For quarks, we assume masses $M_u = M_d = 0$, M_s = 500 MeV, M_c = 1500 MeV, M_b = 5000 MeV, M_t = 35000 MeV. The top quark mass is too large to be relevant, and M_μ, M_d and M_s are light enough that uncertainties in these masses are irrelevant.

The second case for $\sigma_A|v|$ is the cross section expected for annihilation of Majorana neutrinos. It has been emphasized by Goldberg[9] that the annihilation of massive Majorana particles into light fermions is suppressed at low energies because Fermi statistics require the annihilation to be in the p-wave. This suppression will change the Lee-Weinberg bound.[10] The cross section for the annihilation of Majorana neutrinos found from the interaction term

$$\frac{G}{\sqrt{2}} \bar{\nu} \gamma_\mu \gamma_5 \nu \bar{f}_i \gamma_\mu (C_{V_i} - C_{A_i} \gamma_5) f_i , \tag{7}$$

is given by

$$(\sigma_A|v|)_M = \frac{G^2M^2}{2\pi} \sum_i (1-z_i^2)^{1/2} [(C_{V_i}^2 + C_{A_i}^2)2\beta^2/3$$
$$+ C_{A_i}^2 z_i^2/2 + (C_{V_i}^2 - 2C_{A_i}^2)\beta^2 z_i^2/3], \tag{8}$$

where β is the neutrino velocity in the c.m. frame ($\beta^2 \to 3/2x$ in the

N.R. limit).

Equation (4) was integrated with $\sigma_A|v|$ given by Eqs. (6) and (8). For Dirac neutrinos we assumed four neutrino spin degrees of freedom, while for Majorana neutrinos we assumed two. The final value of Y is most conveniently expressed in terms of $\Omega_\nu h^2$[11]

$$\Omega_\nu h^2 = M\ Y\ n_\gamma\ (s/n_\gamma)\rho_c^{-1}$$
$$= 2.7 \times 10^5 (\frac{M}{\mathrm{MeV}})Y, \qquad (9)$$

where as usual Ω_ν is $\rho_\nu \rho_c^{-1}$ ($\rho_c = 3H_o^2/8\pi G$), and $h = H_o/100$ km s^{-1} M_{pc}^{-1}.

Results for $\Omega_\nu h^2$ as a function of M are given in Figure 1 for the Dirac and Majorana case. Although the quantitative results are similar, there are numerous differences in our calculations and previous ones. For instance, Lee and Weinberg assume annihilation through a V-A charged - current Fermi interaction giving $\langle\sigma|v|\rangle = 14G^2M^2/2\pi$. For a mass of 2 GeV we have 5.8 rather than 14 for the annihilation cross section. We will not detail all the differences in our calculation and other similar calculations, but we mention some relevant details we have taken into account: 1) The value of g_* is poorly approximated by a series of step functions in the temperature range 100 - 200 MeV. In the numerical calculations, we have used the calculation of g_* discussed by Olive, Schramm and Steigman.[12] For instance below the quark-hadron transition at T = 200 MeV, $g_* = 69/4$ if we count only the degrees of freedom with M < T, but the actual value of g_* is $\simeq 180/4$ at T = 200 MeV including all the nonrelativistic species. 2) It is necessary to include the term proportional to $C_A^2 z^2$ in $(\sigma_A|v|)_M$, as it can be as large as the $(C_V^2 +$

$C_A^2)\beta^2$ term for certain values of M. 3) The power law approximations for $\Omega(M) \propto M^{-1.8}$ first given by Lee and Weinberg is only approximate - as seen in Figure 1, there is no single power law behavior for $\Omega_\nu h^2(M)$. 4) Freeze-out is not as sudden as usually assumed, and the definition of a single freeze-out temperature is somewhat ambiguous.

We now turn to the interpretation of the results of the numerical work. A limit on $\Omega_\nu h^2$ implies a limit on M. Unfortunately it is not obvious what limit to take for $\Omega_\nu h^2$! If we take the most conservative bound, $\Omega_\nu h^2$ could be as large as 2. In that case the limit on the neutrino mass is M > 1.3 GeV (M > 4.9 GeV) for the Dirac (Majorana) case. We might also say h = 1/2, $\Omega_\nu = 1$ is the preferred value. In this case the limit on the neutrino mass is M > 3.7 GeV (M > 12.6 GeV) for the Dirac (Majorana) case. If we require the age of the Universe to be greater than 15 billion years and $\Omega_\nu = 1$, then $\Omega_\nu h^2 \leq 0.19$,[13] which results in the limits M > 4.2 GeV (M > 13 GeV) for the Dirac (Majorana) case. Due to the uncertainties in the cosmological parameters Ω and h, it is somewhat inappropriate to quote a single limit on M.

The importance of the Lee-Weinberg bound has led us to calculate numerically the contribution of a massive neutrino to $\Omega_\nu h^2$. Previous calculations[2,3,4,5,6,10] have not been as detailed as the calculations reported here. The Lee-Weinberg bound usually quoted as 2 GeV may be somewhere in the region 1.3 - 13 GeV.

References

1. In this paper we will only consider stable (lifetime in excess of the age of the Universe) neutrinos, with masses in excess of a few

MeV, and with annihilations mediated by the Z-bosons of the standard Weinberg-Salam model.

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12. K. A. Olive, D. N. Schramm, and G. Steigman, Nucl. Phys. B180, 497 (1981).
13. This requires $H_o \leq 44$ km s^{-1} Mpc^{-1}, which is within 3σ of the value quoted by A. Sandage and G. A. Tamman, Ap. J. 256, 339 (1982).

Figure Caption

Figure 1: The contribution to $\Omega_\nu h^2$ as a function of M for Dirac and Majorana neutrinos.